Hong Kong Mathematics Olympiad (2010 / 2011)

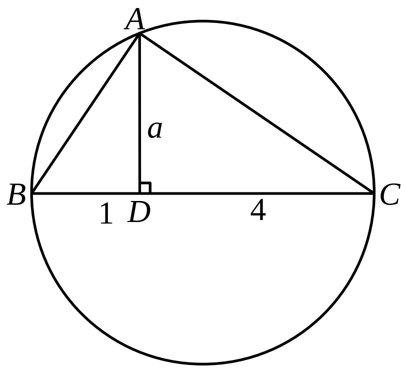
Final Event 1 (Group)

香港数学竞赛 (2010 / 2011)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 在图一中，BC 为圆的直径，A 为圆上的一点，AB、AC 及 AD 为线段，而且 AD 垂直 BC 。若 $BD = 1$，$DC = 4$ 及 $AD = a$ ，求 a 的值。
In Figure 1, BC is the diameter of the circle, A is a point on the circle, AB and AC are line segments and AD is a line segment perpendicular to BC. If $BD = 1$, $DC = 4$ and $AD = a$, find the value of a.



图一
Figure 1

2. 若

$$b = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{-\frac{1}{2}}}}}$$

求 b 的值。

If

$$b = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{-\frac{1}{2}}}}}$$

find the value of b.

3. 若 x、y 及 z 为实数， $xyz \neq 0$， $2xy = 3yz = 5xz$，及 $c = \frac{x+3y-3z}{x+3y-6z}$。求 c 的值。

If x, y and z are real numbers, $xyz \neq 0$, $2xy = 3yz = 5xz$, and $c = \frac{x+3y-3z}{x+3y-6z}$, find the value of c.

4. 若 x 为一整数满足 $\log_{\frac{1}{4}}(2x+1) < \log_{\frac{1}{2}}(x-1)$，求 x 的最大值。

If x is an integer satisfying $\log_{\frac{1}{4}}(2x+1) < \log_{\frac{1}{2}}(x-1)$, find the maximum value of x.

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Final Event 2 (Group)

香港数学竞赛 (2010 / 2011)

决赛项目 2 (团体)

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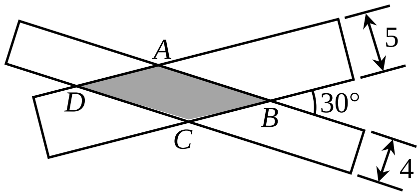
1. 在图一中, 两阔度为 4 及 5 单位的长方形间的夹角为 30° 。求重迭部份的面积。

   In Figure 1, two rectangles with widths 4 and 5 units cross each other at $30°$. Find the area of the overlapped region.



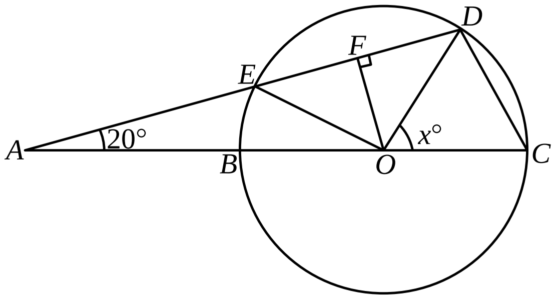
图一
Figure 1

2. 从 1 到 100 选取两数(容许重复) 其和大于 100 。问可选得多少对?

   From 1 to 100, take a pair of numbers (repetitions allowed) so that their sum is greater than 100. How many ways are there to pick such pairs?

3. 在图二中的圆，其圆心为 O 及半径为 r。三角形 ACD 与圆相交于 B、C、D 及 E 点。线段 AE 的长度与圆的半径相同。若 $\angle DAC = 20°$ 及 $\angle DOC = x°$，求 x 的值。

In Figure 2, there is a circle with centre O and radius r. Triangle ACD intersects the circle at B, C, D and E. Line segment AE has the same length as the radius. If $\angle DAC = 20°$ and $\angle DOC = x°$, find the value of x.



图二

Figure 2

4. 已知 $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0$ 及 $\frac{1}{x}-\frac{6}{y}-\frac{5}{z}=0$。

若 $P=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$，求 P 的值。

Given that $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0$ and $\frac{1}{x}-\frac{6}{y}-\frac{5}{z}=0$.

If $P=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$, find the value of P.

Hong Kong Mathematics Olympiad (2010 / 2011)

Final Event 3 (Group)

香港数学竞赛 (2010 / 2011)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 a 为一整数及 a^2+100a 为一质数，求 a 的最大值。

   If a is an integer and a^2+100a is a prime number, find the maximum value of a.

2. 设 a、b 及 c 为实数。若 1 为 $x^2+ax+2=0$ 的根及 a 和 b 为 $x^2+5x+c=0$ 的根，求 $a+b+c$ 的值。

   Let a, b and c be real numbers. If 1 is a root of $x^2+ax+2=0$, and a and b are roots of $x^2+5x+c=0$, find the value of $a+b+c$.

3. 设 x 及 y 为正实数且 $x<y$。若 $\sqrt{x}+\sqrt{y}=1$ 及 $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$，求 $y-x$ 的值。

   Let x and y be positive real number with $x<y$. If $\sqrt{x}+\sqrt{y}=1$ and $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$, find the value of $y-x$.

4. 把数字 $1, 2, \ldots, 10$ 分成两组并设 P_1 及 P_2 分别为该两组数的乘积。若 P_1 为 P_2 的倍数，求 $\frac{P_1}{P_2}$ 的最小值。

Split the numbers $1, 2, 3, \ldots, 10$ into two groups and let P_1 be the product of the first group and P_2 the product of the second group. If P_1 is a multiple of P_2, find the minimum value of $\frac{P_1}{P_2}$.

Hong Kong Mathematics Olympiad (2010 / 2011)

Final Event 4 (Group)

香港数学竞赛 (2010 / 2011)

决赛项目 4 (团体)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $P=2\sqrt[4]{2007\cdot 2009\cdot 2011\cdot 2013+10\cdot 2010\cdot 2010-9}-4000$，求 P 的值。

If $P=2\sqrt[4]{2007\cdot 2009\cdot 2011\cdot 2013+10\cdot 2010\cdot 2010-9}-4000$, find the value of P.

2. 若 $9x^2+nx+1$ 及 $4y^2+12y+m$ 为平方数及 $n>0$，求 $\frac{n}{m}$ 的值。

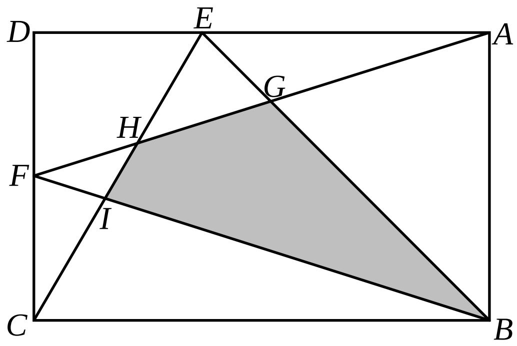
If $9x^2+nx+1$ and $4y^2+12y+m$ are squares with $n>0$, find the value of $\frac{n}{m}$.

3. 设 n 及 $\frac{47}{5}\left(\frac{4}{47}+\frac{n}{141}\right)$ 为正整数。若 r 为 n 被 15 除的余数，求 r 的值。

Let n and $\frac{47}{5}\left(\frac{4}{47}+\frac{n}{141}\right)$ be positive integers. If r is the remainder of n divided by 15, find the value of r.

4. 在图一中，$ABCD$ 为一长方形，及 E 及 F 分别为线段 AD 及 DC 上的点。点 G 为线段 AF 及 BE 的交点，点 H 为线段 AF 及 CE 的交点，点 I 为线段 BF 及 CE 的交点。若 AGE，$DEHF$ 及 CIF 的面积分别为 2，3 及 1，求灰色部份 $BGHI$ 的面积。

In figure 1, $ABCD$ is a rectangle, and E and F are points on AD and DC respectively. Also, G is the intersection of AF and BE, H is the intersection of AF and CE, and I is the intersection of BF and CE. If the areas of AGE, $DEHF$ and CIF are 2, 3 and 1 respectively, find the area of the gray region $BGHI$.



图一

Figure 1